Girraween High School

2022 Year 12 Mathematics Advanced

General Instructions

- Reading Time - 10 minutes
- Working Time - 3 hours
- Write using black pen only
- All questions are compulsory
- Calculators approved by NESA may be used
- Mathematics reference sheets are provided
- Marks may be deducted for careless or badly arranged work

Total Marks:
100

Section I - 10 marks (pages 1-3)

- Attempt questions 1-10, color the bubble next to the letter corresponding to the correct response on your paper
- Allow about 15 minutes for this section

Section II - 90 marks (pages 5-20)

- Attempt questions 11-34
- Allow about 2 hours and 45 minutes for this section

Year 12 Trial HSC Examination - Mathematics 2022

Multiple Choice Answer Sheet

Student Number: ______________ Teacher: ________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct

A ⊗ B ⊗ C ○ D ○

1.	A	○	B	○	C	○	D	○
2.	A	○	B	○	C	○	D	○
3.	A	○	B	○	C	○	D	○
4.	A	○	B	○	C	○	D	○
5.	A	○	B	○	C	○	D	○
6.	A	○	B	○	C	○	D	○
7.	A	○	B	○	C	○	D	○
8.	A	○	B	○	C	○	D	○
9.	A	○	B	○	C	○	D	○
10.	A	○	B	○	C	○	D	○

Question 1 (1 mark)

The midpoint of the line joining $(0, -5)$ to $(d, 0)$ is

A. $\left(\dfrac{d-5}{2}, 0\right)$ B. $\left(0, \dfrac{5-d}{2}\right)$ C. $\left(\dfrac{d}{2}, \dfrac{-5}{2}\right)$ D. $\left(\dfrac{5+d}{2}, 0\right)$

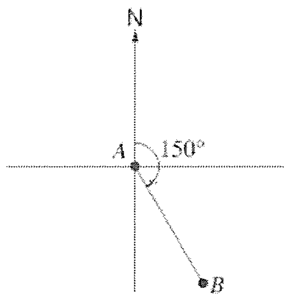
Question 2 (1 mark)

Let $f(x)$ and $g(x)$ be functions such that $f(2) = 5$, $f(3) = 4$, $g(2) = 5$, $g(3) = 2$ and $g(4) = 1$. The value of $f(g(3))$ is:

A. 1 B. 2 C. 4 D. 5

Question 3 (1 mark)

The plane flies on a bearing of 150^o from A to B.



NOT TO SCALE

What is the bearing of A from B?

A. 30^o B. 150^o C. 210^o D. 330^o

Question 4 (1 mark)

A discrete random variable X has a probability distribution as shown:

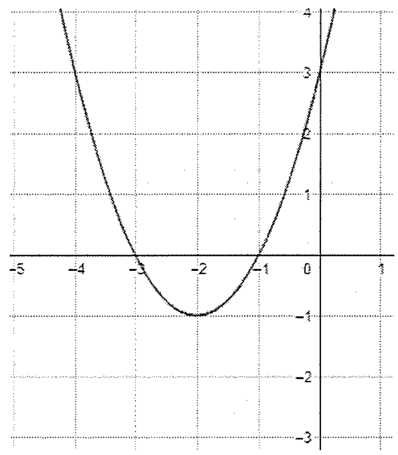
x	0	1	2	3
$\Pr(X = x)$	0.4	0.2	0.3	0.1

The median of X is:

A. 0 B. 1 C. 1.1 D. 2

Question 5 (1 mark)

The graph of the curve $y = (x+a)^2 + b$ is shown below:



What are the values of a and b?

A. $a = 2$, $b = 1$ B. $a = -2$, $b = -1$ C. $a = -2$, $b = 1$ D. $a = 2$, $b = -1$

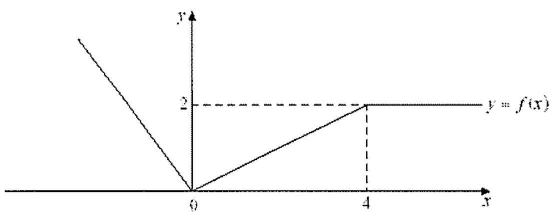
Question 6 (1 mark)

Let $a = e^x$ which expression is equal to $log_e(a^2)$?

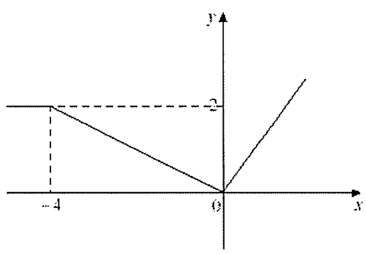
A. e^{2x} B. e^{x^2} C. $2x$ D. x^2

Question 7 (1 mark)

The graph of $y = f(x)$ is shown below:



The graph is transformed to give this graph:

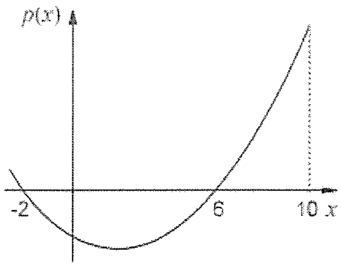


Which of these expressions is the equation of the new graph?

A. $f(x) - 1$ B. $f(-x)$ C. $f(x) + 2$ D. $-f(x)$

Question 8 (1 mark)

The graph shows the function $p(x)$.



If $\int_{-2}^{6} p(x)dx = -10$ and $\int_{-2}^{10} p(x)dx = 1$. What is the value of $\int_{6}^{10} P(x)dx$?

A. 11 B. 9 C. 10 D. 1

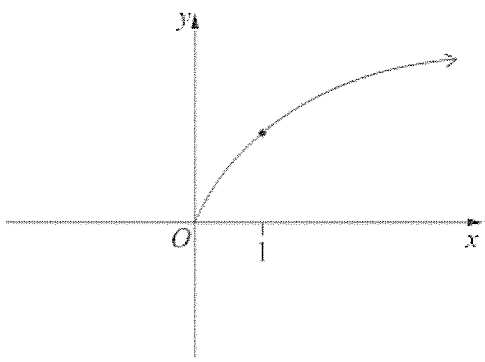
Question 9 (1 mark)

Which inequality defines the domain of the function $f(x) = \dfrac{1}{\sqrt{x+3}}$

A. $x > -3$ B. $x \geq -3$ C. $x < -3$ D. $x \leq -3$

Question 10 (1 mark)

The graph of $y = f(x)$ is shown:



Which of the following inequalities is correct?

A. $f''(1) < 0 < f'(1) < f(1)$

B. $f''(1) < 0 < f(1) < f'(1)$

C. $0 < f''(1) < f'(1) < f(1)$

D. $0 < f''(1) < f(1) < f'(1)$

Mathematics Advanced
Section II Answer Booklet

90 Marks
Attempt Questions 11-34
Allow about 2 hours and 45 minutes for this section

Instructions	• Answer the questions in the spaces provided. • Your responses should include relevant mathematics reasoning and/ or calculations

Please turn over

Question 11 (3 marks)

Solve $3^{2x-1} = 5^x$, giving your answer to 2 d.p.

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Question 12 (3 marks)

(a) Differentiate $3 + sin2x$. [1]

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(b) Hence, or otherwise, find $\int \frac{cos2x}{3 + sin2x} dx$ [2]

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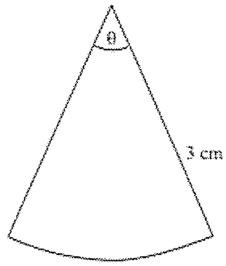
Question 13 (5 marks)



(a) A silver pendant is made in the form of a sector of a circle as shown above. If the radius is 3 cm, what is the angle θ, in radians so that the area is 6 cm^2? [2]

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(b) Another pendant has the same total perimeter, but with radius 2.5 cm. What is the required angle θ, in radians? [3]

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Question 14 (2 marks)

Given that $\int_0^6 (x+k)dx = 30$, and k is a constant, find the value of k.

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Question 15 (7 marks)

Consider the function $f(x) = x^3 - 3x^2$

(a) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. [3]

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(b) Sketch the curve showing where it meets the axes. Label all its important features. You do not need to find points of inflexion. [2]

(c) Find the values of x for which the curve $y = f(x)$ is concave up. [2]

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Question 16 (2 marks)

Evaluate $\int_{e}^{e^3} \frac{5}{x} dx$

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Question 17 (2 marks)

Differentiate $y = x^2 log_e x$

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Question 18 (4 marks)

Class 7C has 18 boys and 12 girls in it and 7K is made up of 12 boys and 16 girls. If you pick one of their class and a pupil from it at random, what is the probability that you select

(a) a girl? [2]

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(b) What is the probability that the student is from 7C given a girl is selected? [2]

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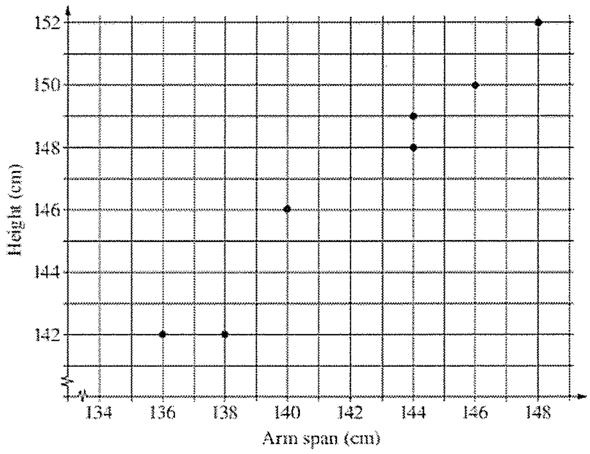
Question 19 (5 marks)

A set of bivariate data is collected by measuring the height and arm span of seven children. The graph shows a scatterplot of these measurements.



(a) Complete the table below. [1]

Arm Span (cm)							
Height (cm)							

(b) Calculate Pearson's correlation coefficient for the data, correct to two decimal places. [1]

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(c) Identify the direction and the strength of the linear association between height and arm span. [2]

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(d) The equation of the least-squares regression line is shown. [1]

Height=0.866 $\times$ (arm span)+23.7 A child has an arm span of 143 cm.

Calculate the predicted height for this child using the equation of the least-squares regression line.

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Question 20 (2 marks)

Prove that $\dfrac{1 - sin^2x cos^2x}{sin^2x} = cot^2x + sin^2x$

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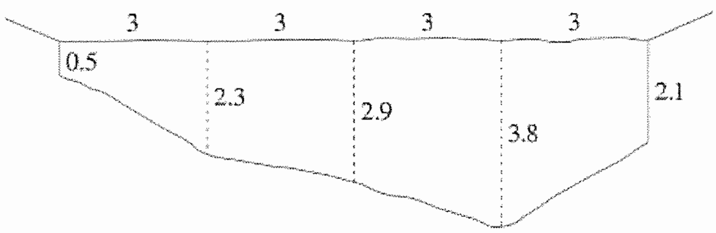
Question 21 (3 marks)

At a certain location a river is 12 metres wide. At this location the depth of the river, in metres, has been measured at 3 metre intervals. The cross-section is shown below.



Use the Trapeziodal rule with the five depth measurements to calculate the approximate area of the cross-section.

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Question 22 (7 marks)

The velocity of a particle moving along the x–axis is given by:

$$\dot{x} = 8 - 8e^{-2t}$$

where t is the time in seconds and x is the displacement in metres.

(a) Show that the particle is initially at rest. [1]

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(b) Show that the acceleration of the particle is always positive. [1]

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(c) Explain why the particle is moving in the positive direction for all $t > 0$. [2]

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(d) As $t \to \infty$, the velocity of the particle approaches a constant. Find the value of the constant. [1]

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(e) Sketch the graph of the particle's velocity as a function of time. [2]

Question 23 (4 marks)

The circle $x^2 + 6x + y^2 + 2y = -6$ is reflected in the y-axis.Sketch the reflected circle, showing the coordinates of the centre and the radius.

Question 24 (2 marks)

A town planning committee notes that the rate of growth of the town's population since 1985 has followed the formula: $\dfrac{dp}{dt} = (1500 + 200t)$ people per year where t is the number of years since 1st January 1985. On 1st January 1992 the population was 25 000. Find the formula for the city's population from 1985 onwards.

Question 25 (4 marks)

Solve $10sin^2\, x - 4sin\, x = 5$ for $0^o \leq x \leq 360^o$.

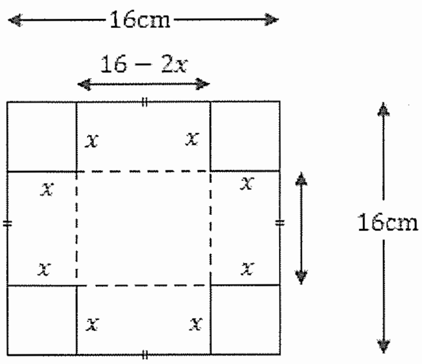
Question 26 (4 marks)

(a) Differentiate $log_e(cos\; x)$ with respect to x. [2]

(b) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{4}} tan\; x\; dx$ [2]

Question 27 (6 marks)

A square sheet of metal is 16cm on each side. Squares of side x cm are cut from each corner. The sheet is then bent along the dotted lines to form an open box.



(a) Show that the volume of the box V, expressed as a function of x is given by: $V(x) = 4x(x-8)^2$ [2]

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(b) Hence find the maximum volume of the box, and the value of x at which it occurs. [4]

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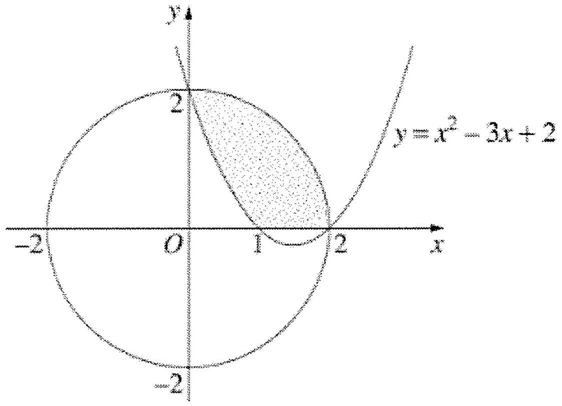
Question 28 (3 marks)



The shaded region in the diagram is bounded by the circle of radius 2 centred at the origin, the parabola $y = x^2 - 3x + 2$, and the x-axis.

By considering the difference of two areas, find the area of the shaded region.

Question 29 (6 marks)

In a game a six-sided dice is rolled twice.

The difference between the two results is recorded.

(a) The following table is produced. Some of the results are missing. Complete the table. [1]

		Second roll					
		1	**2**	**3**	**4**	**5**	**6**
First roll	**1**	0	1	2	3		5
	2	1	0	1	2	3	
	3	2	1			2	3
	4		2	1	0	1	
	5	4	3	2	1	0	1
	6	5	4	3	2		0

(b) A probability distribution table is drawn to summarise the results. Complete the table [2]

x	0	1	2	3	4	5
$P(X=x)$			$\frac{2}{9}$			$\frac{1}{18}$

(c) Find the expected value, $E(X)$, and the standard deviation σ. [3]

Question 30 (3 marks)

The sum of the series $1 + 8 + 15 + \ldots$ is 396. How many terms does the series contain?

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Question 31 (2 marks)

A circular clock face, centre O, has a minute hand OA and an hour hand OB.

$OA = 10\ cm$, $OB = 7\ cm$.

Calculate the length of AB when the hands show 5 o'clock.

Give your answer correct to 1 decimal place.

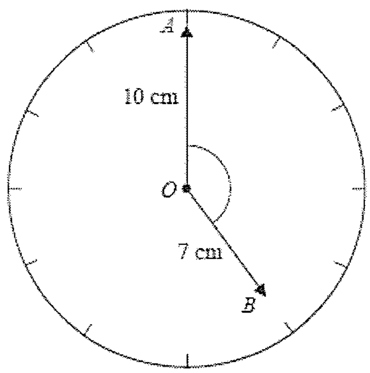

Diagram NOT accurately drawn

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Question 32 (1 mark)

Evaluate $ln\, 3$ correct to three significant figures.

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Question 33 (4 marks)

By considering the equation of the tangent to $y = x^2 - 1$ at the point $(a, a^2 - 1)$, find the equations of the two tangents to $y = x^2 - 1$ which pass through $(3, -8)$.

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Question 34 (6 marks)

Under certain climatic conditions the number N of blue-green algae satisfies the equation

$$N(t) = Ae^{0.15t}$$

where t is measured in days and A is a constant.

(a) Show that the number of algae increases at a rate proportional to the number. [2]

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(b) When $t = 3$ the number of algae was estimated to be 1.7×10^8. Evaluate A. [2]

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(c) The number of algae doubles every x days. Find the value of x. [2]

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End of exam

2022 - HSC Trials - Solutions
Mathematics Advanced

Question 1 (1 mark)

The midpoint of the line joining $(0, -5)$ to $(d, 0)$ is

A. $\left(\frac{d-5}{2}, 0\right)$ B. $\left(0, \frac{5-d}{2}\right)$ (C.) $\left(\frac{d}{2}, \frac{-5}{2}\right)$ D. $\left(\frac{5+d}{2}, 0\right)$

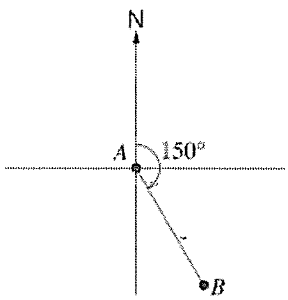
Question 2 (1 mark)

Let $f(x)$ and $g(x)$ be functions such that $f(2) = 5$, $f(3) = 4$, $g(2) = 5$, $g(3) = 2$ and $g(4) = 1$. The value of $f(g(3))$ is:

A. 1 B. 2 C. 4 (D.) 5

Question 3 (1 mark)

The plane flies on a bearing of 150^o from A to B.



What is the bearing of A from B?

A. 30^o B. 150^o C. 210^o (D.) 330^o

Question 4 (1 mark)

A discrete random variable X has a probability distribution as shown:

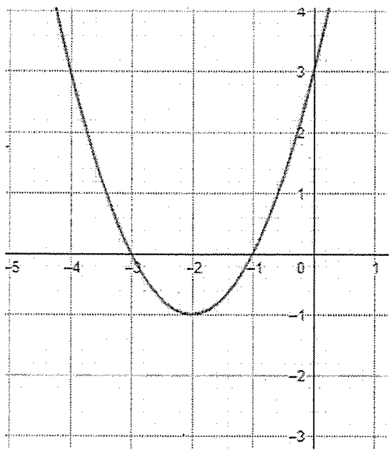
x	0	1	2	3
$\Pr(X = x)$	0.4	0.2	0.3	0.1

The median of X is:

A. 0 (B.) 1 C. 1.1 D. 2

Question 5 (1 mark)

The graph of the curve $y = (x + a)^2 + b$ is shown below:



What are the values of a and b?

A. $a = 2, b = 1$ B. $a = -2, b = -1$ C. $a = -2, b = 1$ D. $a = 2, b = -1$

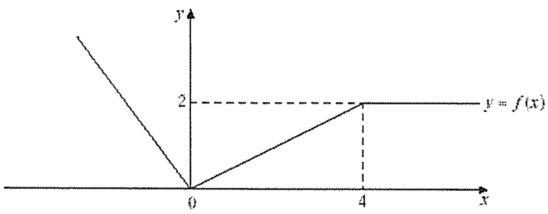
Question 6 (1 mark)

Let $a = e^x$ which expression is equal to $log_e(a^2)$?

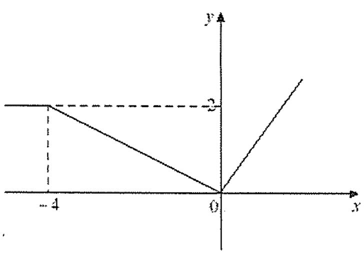
A. e^{2x} B. e^{x^2} C. $2x$ D. x^2

Question 7 (1 mark)

The graph of $y = f(x)$ is shown below:



The graph is transformed to give this graph:

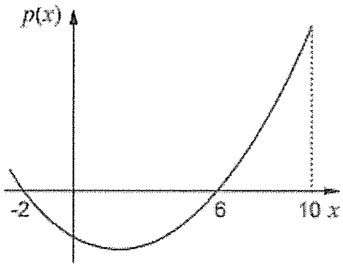


Which of these expressions is the equation of the new graph?

A. $f(x) - 1$ B. $f(-x)$ C. $f(x) + 2$ D. $-f(x)$

Question 8 (1 mark)

The graph shows the function $p(x)$.



If $\int_{-2}^{6} p(x)dx = -10$ and $\int_{-2}^{10} p(x)dx = 1$. What is the value of $\int_{6}^{10} P(x)dx$?

A. 11   B. 9   C. 10   D. 1

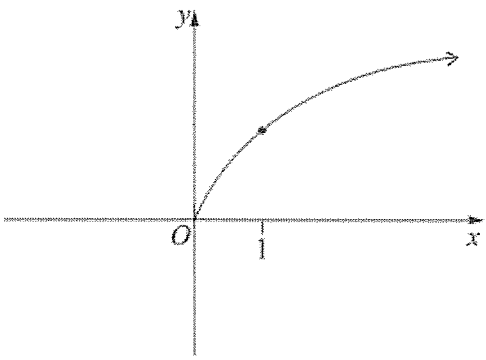
Question 9 (1 mark)

Which inequality defines the domain of the function $f(x) = \dfrac{1}{\sqrt{x+3}}$

A. $x > -3$ B. $x \geq -3$ C. $x < -3$ D. $x \leq -3$

Question 10 (1 mark)

The graph of $y = f(x)$ is shown:



Which of the following inequalities is correct?

A. $f''(1) < 0 < f'(1) < f(1)$

B. $f''(1) < 0 < f(1) < f'(1)$

C. $0 < f''(1) < f'(1) < f(1)$

D. $0 < f''(1) < f(1) < f'(1)$

Question 11 (3 marks)

Solve $3^{2x-1} = 5^x$, giving your answer to 2 d.p.

$3^{2x-1} = 5^x$

Applying ln on both sides

$\ln(3^{2x-1}) = \ln 5^x$

$(2x-1)\ln 3 = x\ln 5$

$2x\ln 3 - \ln 3 = x\ln 5$

$x(2\ln 3 - \ln 5) = \ln 3$

$\Rightarrow x = \frac{\ln 3}{2\ln 3 - \ln 5} \simeq 1.87$ (to 2 d.p.)

Question 12 (3 marks)

(a) Differentiate $3 + sin2x$. [1]

$y = 3 + \sin 2x$

$\frac{dy}{dx} = 2\cos 2x$

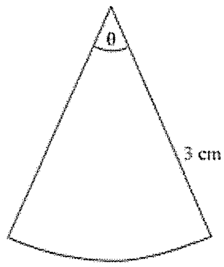
(b) Hence, or otherwise, find $\int \frac{cos2x}{3 + sin2x} dx$ [2]

$\int \frac{\cos 2x}{3+\sin 2x} dx$

$= \frac{1}{2}\int \frac{2\cos 2x}{3+\sin 2x} dx$

$= \frac{1}{2}\ln|3+\sin 2x| + C$ from part a

Question 13 (5 marks)



(a) A silver pendant is made in the form of a sector of a circle as shown above. If the radius is 3 *cm*, what is the angle θ, in radians so that the area is 6 cm^2? [2]

Sector area = 6

$= \frac{1}{2} \times 3^2 \times \theta$

$\Rightarrow \theta = \frac{4}{3}^c$

(b) Another pendant has the same total perimeter, but with radius 2.5 *cm*. What is the required angle θ, in radians? [3]

Total perimeter $= 3 + 3 + (3 \times \frac{4}{3})$

$= 10\,cm$

Hence, for the second pendant

$10 = 2.5 + 2.5 + 2.5 \times \theta$

$\Rightarrow \theta = 2^c$

Question 14 (2 marks)

Given that $\int_0^6 (x + k)dx = 30$, and k is a constant, find the value of k.

$\int_0^6 (x + K)\,dx = 30$

$\left[\frac{x^2}{2} + Kx\right]_0^6 = 30$

$\left(\frac{6^2}{2} + 6K\right) - \left(\frac{0^2}{2} + 0\right) = 30$

$18 + 6K = 30$

$6K = 30 - 18$

$6K = 12$

$K = 2$

Question 15 (7 marks)

Consider the function $f(x) = x^3 - 3x^2$

(a) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. [3]

$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

Stationary points when $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0$ or $x = 2$

when $x = 0$

$f(0) = 0$

$f''(0) = 0 - 6$

$= -6 < 0$

∴ maximum turning point at $(0,0)$

when $x = 2$

$f(2) = 2^3 - (3 \times 4) = -4$

$f''(2) = 6 \times 2 - 6 = 6 > 0$ ∴ minimum turning point at $(2,-4)$

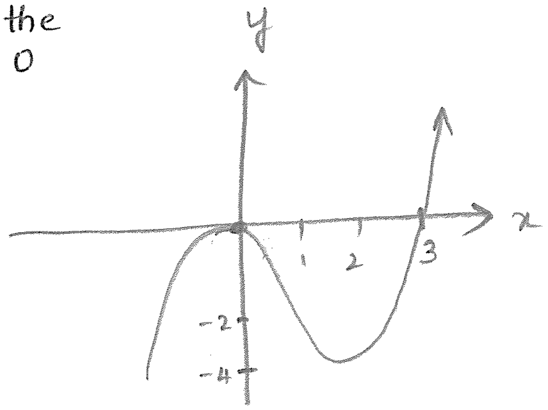
(b) Sketch the curve showing where it meets the axes. Label all its important features. You do not need to find points of inflexion. [2]

$f(x) = x^3 - 3x^2$ meets the x-axis when $f(x) = 0$

$x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$x = 0$ or 3



(c) Find the values of x for which the curve $y = f(x)$ is concave up. [2]

f(x) is concave up when

$f''(x) > 0$

$6x - 6 > 0$ ∴ f(x) is concave up

$6x > 6$ when $x > 1$

$x > 1$

Question 16 (2 marks)

Evaluate $\int_{e}^{e^3} \frac{5}{x} dx$

$\int_{e}^{e^3} \frac{5}{x} dx = 5 \int_{e}^{e^3} \frac{1}{x} dx = 5 [\ln x]_{e}^{e^3}$

$= 5 [\ln e^3 - \ln e]$

$= 5 [3 \ln e - 1]$

$= 5 [3 - 1]$

$= 10$

Question 17 (2 marks)

Differentiate $y = x^2 log_e x$

$y = x^2 \log_e x$

using product rule

$y' = u'v + uv'$

$y' = x^2 \cdot \frac{1}{x} + 2x \cdot \log_e x$

$y' = x + 2x \ln x$

Question 18 (4 marks)

Class 7C has 18 boys and 12 girls in it and 7K is made up of 12 boys and 16 girls. If you pick one of their class and a pupil from it at random, what is the probability that you select

(a) a girl? [2]

$p(girl) = \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{4}{7}$

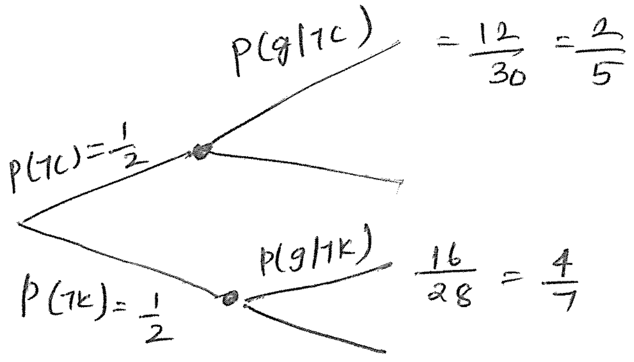
$= \frac{1}{5} + \frac{2}{7}$

$= \frac{17}{35}$

(b) What is the probability that the student is from 7C given a girl is selected? [2]

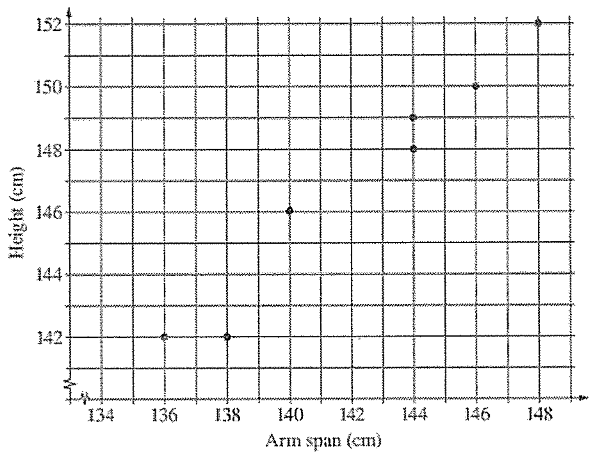
$p(7C|girl) = \frac{p(7C \cap girl)}{p(girl)} = \frac{\left(\frac{1}{2} \times \frac{2}{5}\right)}{\left(\frac{17}{35}\right)}$

$= \frac{7}{17}$



Question 19 (5 marks)

A set of bivariate data is collected by measuring the height and arm span of seven children. The graph shows a scatterplot of these measurements.



(a) Complete the table below. [1]

Arm Span (cm)	136	138	140	144	144	146	148
Height (cm)	142	142	146	148	149	150	152

(b) Calculate Pearson's correlation coefficient for the data, correct to two decimal places. [1]

0·98 (2d·p)

(c) Identify the direction and the strength of the linear association between height and arm span. [2]

positive, strong

(d) The equation of the least-squares regression line is shown. [1]

Height=0.866 × (arm span)+23.7 A child has an arm span of 143 cm.

Calculate the predicted height for this child using the equation of the least-squares regression line.

0·866 X 143 + 23·7

= 147·538 Cm

Question 20 (2 marks)

Prove that $\dfrac{1 - sin^2x cos^2x}{sin^2x} = cot^2x + sin^2x$

L.H.S   R.H.S

R.H.S: $\cot^2 x + \sin^2 x$

$= \dfrac{\cos^2 x}{\sin^2 x} + \sin^2 x$

$= \dfrac{\cos^2 x + \sin^4 x}{\sin^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x \cdot \sin^2 x}{\sin^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x(1 - \cos^2 x)}{\sin^2 x}$

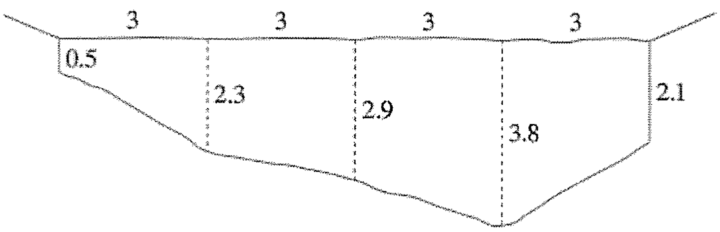
$= \dfrac{\cos^2 x + \sin^2 x - \sin^2 x \cos^2 x}{\sin^2 x}$

$= \dfrac{1 - \sin^2 x \cos^2 x}{\sin^2 x}$

$=$ L.H.S

Question 21 (3 marks)

At a certain location a river is 12 metres wide. At this location the depth of the river, in metres, has been measured at 3 metre intervals. The cross-section is shown below.



x	0	3	6	9	12
h	0.5	2.3	2.9	3.8	2.1

Use the Trapeziodal rule with the five depth measurements to calculate the approximate area of the cross-section.

$A \simeq \frac{3}{2}\left[0.5 + 2(2.3 + 2.9 + 3.8) + 2.1\right]$

$\simeq \frac{3}{2}(20.6)$

$\simeq 30.9\ m^2$

Question 22 (7 marks)

The velocity of a particle moving along the x−axis is given by:

$$\dot{x} = 8 - 8e^{-2t}$$

where t is the time in seconds and x is the displacement in metres.

(a) Show that the particle is initially at rest. [1]

$\dot{x} = 8 - 8e^{-2t}$

Sub $t = 0$

$\dot{x} = 8 - 8e^{-2(0)}$

$\dot{x} = 8 - 8$

$\dot{x} = 0$

∴ particle initially at rest

(b) Show that the acceleration of the particle is always positive. [1]

$\ddot{x} = 16e^{-2t}$

As $e^{-2t} > 0$, for all values of t, then $\ddot{x}$ is always positive

(c) Explain why the particle is moving in the positive direction for all $t > 0$. [2]

Since particle is initially at rest (from a) and always a positive acceleration is applied (from b) ∴ for all $t > 0$

This means that it is moving in the positive direction.

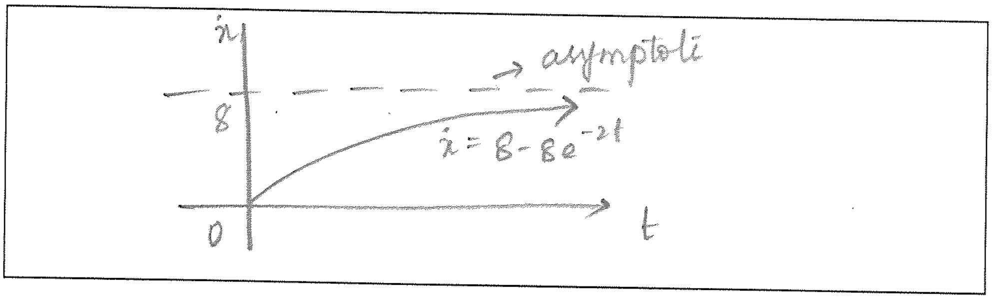
(d) As $t \to \infty$, the velocity of the particle approaches a constant. Find the value of the constant. [1]

$\dot{x} = 8 - 8e^{-2t}$

As $t \to \infty$, $e^{-2t} \to 0$

∴ $\dot{x} \to 8$ ∴ The constant is 8.

(e) Sketch the graph of the particle's velocity as a function of time. [2]



Question 23 (4 marks)

The circle $x^2 + 6x + y^2 + 2y = -6$ is reflected in the y-axis.Sketch the reflected circle, showing the coordinates of the centre and the radius.

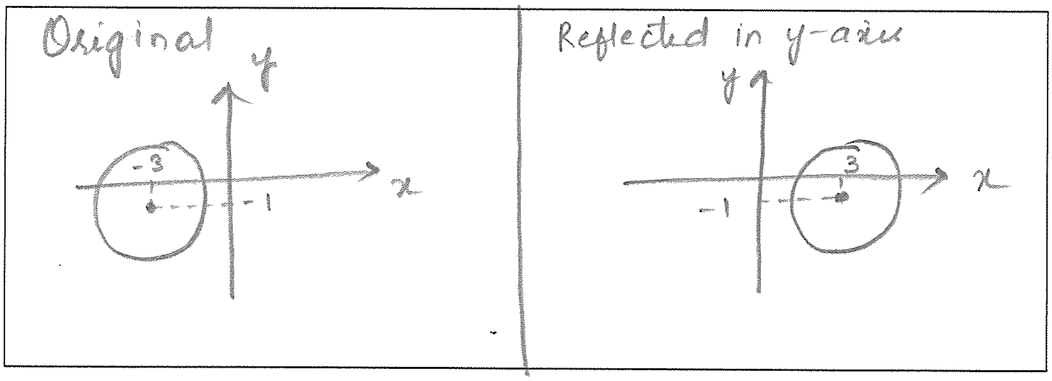
$x^2 + 6x + y^2 + 2y = -6$

$x^2 + 6x + 3^2 + y^2 + 2y + 1^2 = -6 + 9 + 1$

$(x+3)^2 + (y+1)^2 = 4$

$\therefore$ radius = 2 units

centre = $(-3, -1)$



Question 24 (2 marks)

A town planning committee notes that the rate of growth of the town's population since 1985 has followed the formula: $\frac{dp}{dt} = (1500 + 200t)$ people per year

where t is the number of years since 1st January 1985. On 1st January 1992 the population was 25 000. Find the formula for the city's population from 1985 onwards.

population $= \int (1500 + 200t)\, dt$

$= 1500t + \frac{200t^2}{2} + K$

$= 1500t + 100t^2 + K$

when $t = 7$, $p = 25\,000$, putting this information into the formula gives

$25\,000 = 10\,500 + 4900 + K$

$\Rightarrow K = 9600$

population $= 100t^2 + 1500t + 9600$

Question 25 (4 marks)

Solve $10sin^2 x - 4sin\, x = 5$ for $0^o \leq x \leq 360^o$.

$10\sin^2 x - 4\sin x - 5 = 0$

using Quadratic formula

$\sin x = \frac{4 \pm \sqrt{216}}{20}$

$\sin x \simeq 0.93485$ or -0.534847

$x = 69.2^0, 110.8^0$ or $212.3^0, 327.7^0$

Question 26 (4 marks)

(a) Differentiate $log_e(cos\, x)$ with respect to x. [2]

$y = \log_e(\cos x)$

$\frac{dy}{dx} = -\frac{\sin x}{\cos x}$

$= -\tan x$

(b) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{4}} tan\, x\, dx$ [2]

$\int_0^{\frac{\pi}{4}} \tan x \, dx = -\left[\log_e \cos x\right]_0^{\frac{\pi}{4}}$

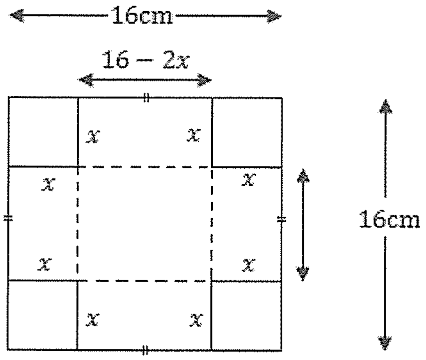
$= -\left[\log_e \cos\frac{\pi}{4} - \log_e \cos 0\right]$

$= -\left[\log_e\left[\frac{1}{\sqrt{2}}\right] - \log_e 1\right]$

$= -\log \frac{1}{\sqrt{2}} \simeq 0.35$ (2 d.p)

Question 27 (6 marks)

A square sheet of metal is 16cm on each side. Squares of side x cm are cut from each corner. The sheet is then bent along the dotted lines to form an open box.



(a) Show that the volume of the box V, expressed as a function of x is given by: $V(x) = 4x(x-8)^2$ [2]

length $= 16-2x$; breadth $= 16-2x$; height $= x$

$\therefore V = (16-2x)^2 x$

$V(x) = x(2^2(8-x)^2)$

$\therefore V(x) = 4x(8-x)^2$

(b) Hence find the maximum volume of the box, and the value of x at which it occurs. [4]

$V(x) = 4x(8-x)^2$

$V'(x) = 4(x-8)^2 + 8x(x-8)$

$V'(x) = 4(x-8)\,[(x-8)+2x]$

$V'(x) = 4(x-8)(3x-8)$

Finding stationary points

sub $V'(x) = 0$

$4(x-8)(3x-8) = 0$

$x = 8$; $x = \frac{8}{3}$

$V''(x) = 24x - 128$

when $x = 8$

$V''(8) = 24(8) - 128$

> 0

minimum turning point

when $x = \frac{8}{3}$

$V''\left(\frac{8}{3}\right) = 24\left(\frac{8}{3}\right) - 128$

< 0

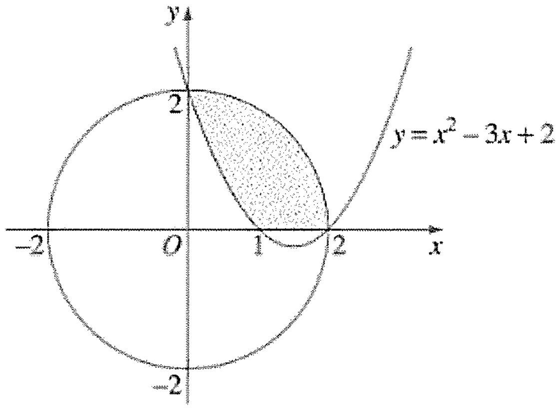
maximum turning point

$\therefore$ The maximum volume when $x = \frac{8}{3}$ is

$V\left(\frac{8}{3}\right) = 4 \times \frac{8}{3}\left(\frac{8}{3} - 8\right)^2$

$= 330\frac{11}{27}$ cm^3

Question 28 (3 marks)



The shaded region in the diagram is bounded by the circle of radius 2 centred at the origin, the parabola $y = x^2 - 3x + 2$, and the x-axis.

By considering the difference of two areas, find the area of the shaded region.

Shaded area = area in the Quarter circle less the area below the parabola between $x = 0$ & 1

Area of $\frac{1}{4}$ circle $= \frac{1}{4}\pi r^2$

$= \frac{1}{4}\pi(2)^2$

$= \pi$ sq units

Area of the parabola between $x=0$ & $x=1$

$= \int_0^1 y\,dx$

$= \int_0^1 (x^2 - 3x + 2)\,dx$

$= \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^1 = \left(\left(\frac{1}{3} - \frac{3}{2} + 2\right) - 0\right)$

$= \frac{5}{6}$

$\therefore$ Shaded area $= \left(\pi - \frac{5}{6}\right) u^2$

Question 29 (6 marks)

In a game a six-sided dice is rolled twice.

The difference between the two results is recorded.

(a) The following table is produced. Some of the results are missing. Complete the table. [1]

		Second roll					
		1	**2**	**3**	**4**	**5**	**6**
First roll	**1**	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

(b) A probability distribution table is drawn to summarise the results. Complete the table [2]

x	0	1	2	3	4	5
$P(X=x)$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

(c) Find the expected value, $E(X)$, and the standard deviation σ. [3]

$E(X) = 0\left(\frac{1}{6}\right) + 1\left(\frac{5}{18}\right) + 2\left(\frac{2}{9}\right) + 3\left(\frac{1}{6}\right)$

$+ 4\left(\frac{1}{9}\right) + 5\left(\frac{1}{18}\right)$

$= \frac{35}{18} = 1\frac{17}{18}$

$Var(X) = E(X^2) - \mu^2$

$\sigma = \sqrt{Var(X)}$

$\therefore E(X^2) = 0^2\left(\frac{1}{6}\right) + 1^2\left(\frac{5}{18}\right) + 2^2\left(\frac{2}{9}\right) + 3^2\left(\frac{1}{6}\right) + 4^2\left(\frac{1}{9}\right) + 5^2\left(\frac{1}{18}\right)$

$= 5\frac{5}{6}$

As $E(X) = \mu = 1\frac{17}{18}$ then $\mu^2 = 3\frac{253}{324}$

$= 5\frac{5}{6} - 3\frac{253}{324}$

$= 2.0524$

$\sigma = 1.43$ (2d.p)

Question 30 (3 marks)

The sum of the series $1 + 8 + 15 + \ldots$ is 396. How many terms does the series contain?

This is an arithmetic sequence with first term 1 and common difference 7.
Let the number of terms in the sequence be n.

$\therefore S_n = 396$

$\Rightarrow \frac{n}{2}(2+7(n-1)) = 396$

$n(7n-5) = 792$

$7n^2 - 5n - 792 = 0$

$(7n+72)(n-11) = 0$

$\Rightarrow n = 11$

Since $-\frac{72}{7}$ is not an integer

$\therefore$ The number of terms are 11.

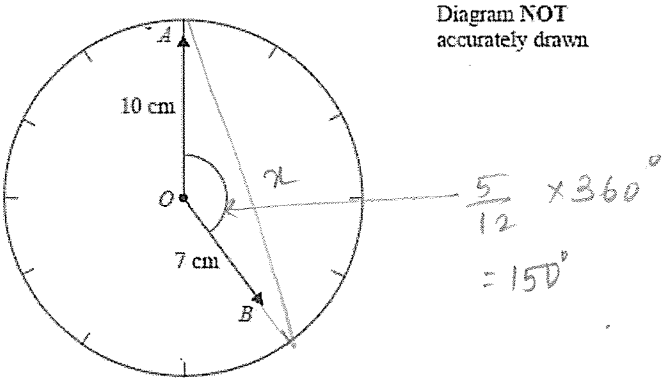
Question 31 (2 marks)

A circular clock face, centre O, has a minute hand OA and an hour hand OB.

$OA = 10\ cm$, $OB = 7\ cm$.

Calculate the length of AB when the hands show 5 o'clock.

Give your answer correct to 1 decimal place.



Using cosine rule

$x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 150^\circ$

$x^2 = 270.243$

$x \simeq \sqrt{270.243}$

$x \simeq 16.439$

$x \simeq 16.4\ cm$ (3 sig. fig.; correct to 1 d.p)

Question 32 (1 mark)

Evaluate $ln\, 3$ correct to three significant figures.

$\ln 3 = 1.10$ (3 sig fig)

Question 33 (4 marks)

By considering the equation of the tangent to $y = x^2 - 1$ at the point $(a, a^2 - 1)$, find the equations of the two tangents to $y = x^2 - 1$ which pass through $(3, -8)$.

$y = x^2 - 1$

$y' = 2x$

At $x = a$; $y' = 2a$

Equation of the line with gradient $2a$, through $(a, a^2 - 1)$

$y - (a^2 - 1) = 2a(x - a)$

$y - a^2 + 1 = 2ax - 2a^2$

$y = 2ax - a^2 - 1$

If the tangent passes through $(3, -8)$

$2a(3) - a^2 - 1 = -8$

$6a - a^2 - 1 = -8$

$a^2 - 6a - 7 = 0$

$(a - 7)(a + 1) = 0$

$\Rightarrow a = 7$ or $a = -1$

$\therefore$ Equations of tangents are

$y = 14x - 50$

$y = -2x - 2$

Question 34 (6 marks)

Under certain climatic conditions the number N of blue-green algae satisfies the equation

$$N(t) = Ae^{0.15t}$$

where t is measured in days and A is a constant.

(a) Show that the number of algae increases at a rate proportional to the number. [2]

$N(t) = Ae^{0.15t}$

$N'(t) = 0.15Ae^{0.15t}$

$= 0.15N(t)$

$N'(t) \propto N(t)$

(b) When $t = 3$ the number of algae was estimated to be 1.7×10^8. Evaluate A. [2]

$1.7 \times 10^8 = Ae^{0.15(3)}$

$1.7 \times 10^8 = Ae^{0.45}$

$A = \dfrac{1.7 \times 10^8}{e^{0.45}}$

$= 1.0839 \times 10^8$

$= 1.1 \times 10^8$ (2 sig. fig.)

(c) The number of algae doubles every x days. Find the value of x. [2]

Now $N(0) = Ae^{0.15(0)}$

$= A$

$\therefore N(x) = 2A$

$\therefore Ae^{0.15x} = 2A$

$e^{0.15x} = 2$

$0.15x = \ln 2$

$x = \dfrac{\ln 2}{0.15}$

$= 4.6209$

$= 4.6$ days

End of exam